**Title: The Balancing Act**

**Link to Outcomes:**

- **Problem Solving** Students will conduct an experiment which will result in their ability to determine the balance point (center of mass or gravity) of any triangle.
- **Reasoning** Students will apply their discoveries in the investigation of an open-ended extension.
- **Technology** Students will use *The Geometer's Sketchpad* to construct various triangles and quadrilaterals in an attempt to verify hypotheses.
- **Real-World Application** Students will solve a problem that is relevant to architecture and construction.
- **Modeling** Students will model a physical problem using computer simulation.
- **Connections** Students will explore the geometric aspects of a physics application.
- **Cooperation** Students will demonstrate the ability to conduct a joint experiment in small groups and agree on conclusions.
- **Communication** Students will demonstrate the ability to communicate their hypotheses and their conclusions both verbally and in writing.

**Brief Overview:**

This activity consists of four parts. However, it is not necessary to do all parts.

In Part 1, students will become proficient using *The Geometer's Sketchpad* by constructing different types of triangles and quadrilaterals.

Note that students must be familiar with the definitions of different types of quadrilaterals to complete Part 1 of the learning unit. However, Part 1 can be presented either before or after students have formally studied the properties of quadrilaterals. Presenting it after the properties of the quadrilaterals are studied gives students more options in their ability to use *The Geometer's Sketchpad* to do the required constructions.

In Part 2, students will examine various triangles to determine the "balance point" for each. They will use hands-on physical models of triangles and *Sketchpad.* The balance point will be determined by using both a physical model and geometric properties. Students will state a general description for how to determine the balance point for any triangle

In Part 3, students will experiment to address the more open-ended question "Is there a balance point for any given quadrilateral and how can it be found?"

In Part 4, students will investigate relationships between perimeters and areas of triangles formed by using medians of various triangles and the perimeters and areas of the triangles themselves.

**Grade/Level:**

Grades 9-12, Geometry or Geometry Honors

**Duration/Length:**

5 hours

**Prerequisite Knowledge:**

- Some familiarity with *The Geometer's Sketchpad*
- Basic geometric concepts and terms
- Types and properties of triangles and quadrilaterals
- Area

**Objectives:**

- To apply definitions and properties to construct various triangles and quadrilaterals using *The Geometer's Sketchpad*
- To discover a relationship between Geometry and Physics
- To use *The Geometer's Sketchpad* to test hypotheses

**Materials/Resources/Printed Materials:**

- Student Activity Sheet 1 "Constructing Geometric Figures"
- Student Activity Sheet 2 "The Balancing Act Starring…The Triangle."
- Student Activity Sheet 3 "The Balancing Act Starring…The Quadrilateral"
- Cardboard (thin, but not too flexible). Manila folders will serve nicely.
- Rulers
- Scissors
- String

**Development/Procedures:**

**Part 1 (Duration – 90 minutes):**

1. Group students in pairs at the computers
2. Distribute Activity Sheet 1 "Constructing Geometric Figures" to students. Demonstrate the three construction methods that appear at the beginning of the activity sheet (for constructing an equilateral triangle). Have students duplicate these constructions for practice.
3. Review the definitions of the various types of triangles and quadrilaterals. (Additional properties of quadrilaterals may also be reviewed if previously studied.)
4. Groups should perform the remainder of the constructions that are called for on Activity Sheet 1. Three different methods of constructing an equilateral triangle appear on the sheet as models for the students.

5. Have students from each group demonstrate their constructions for the class. Examine different methods for constructing the same figures.

**Part 2 (Duration – 2 hours):**

1. Group students in pairs.
2. Give each group 2 pieces of cardboard.
3. Instruct students to draw a triangle on the cardboard.
4. Have students cut out the cardboard triangle and poke a small hole through it as close to one vertex as possible.
5. Have students thread a piece of string through the hole and hold the string up against the edge of a chalkboard (trying not to let the triangle touch the chalkboard) and let the triangle swing free until it comes to rest. Students should then mark the point on the opposite side of the triangle that is directly (vertically) below the vertex with the string.
6. Have students repeat this process for the other two vertices of the triangle.
7. Students should then draw line segments from each vertex to the point marked on the side opposite it. The three segments should intersect at the same point.
8. Ask the students to make conjectures about the three segments and their point of intersection. Some conjectures might be:
    - The three line segments are medians.
    - The point of intersection of the medians (the term centroid can be introduced here) is twice as far from the vertex as it is from the opposite side.
    - The three line segments each divide the original triangle into two triangles of equal areas (a well known property of medians).
    - If line segments are drawn from the centroid to each vertex, the areas of the three triangles formed are equal.
9. Ask students to use the eraser end of a pencil and try to balance the triangle on the pencil at the point that they obtained.
10. Lead the class through a discussion of center of mass (center of gravity, Centroid) and its implications in physics, architecture and construction. It may be important to indicate that because the cardboard has a negligible thickness, mass and area are interchangeable in this discussion.
11. Distribute Student Activity Sheet 2 "The Balancing Act Starring…The Triangle." This will lead students through a *Sketchpad* activity to verify (or reject) their conjectures.
12. Follow up with a discussion justifying these conjectures.

**Part 3 (Duration – 1 hour):**

Students will now begin an investigation to try to find the "balance point" for various quadrilaterals.

1. Group students in pairs.
2. Distribute Student Activity Sheet 3 "The Balancing Act Starring…The Quadrilateral." This activity sheet will lead students to use *Sketchpad* to formulate hypotheses about where the balance point of a quadrilateral is located.

3. If possible, allow students to print a copy of a quadrilateral they construct (showing their hypothetical balance point). Then, tracing onto cardboard, they can try to verify whether or not they have correctly located the centroid. Note, our method for constructing the centroid of a kite is as follows:
    a. Find point A, the midpoint of the symmetry diagonal.
    b. Draw segments connecting the midpoints of opposite sides of the kite and determine point B, the point of intersection of these segments.
    c. The centroid of the kite is the point P such that the ratio of AP : BP is 2 : 1.

    or

    a. Draw the non-symmetry diagonal, creating two isosceles triangles.
    b. Call the heights of these two triangles $h_1$ and $h_2$, where $h_2 > h_1$.
    c. Locate a point on the symmetry diagonal and inside the larger of the two triangles whose distance from the point of intersection of both diagonals is $1/3(h_2-h_1)$. This point is the centroid of the kite.

    or

    See printouts of student attempts to find the centroid of a kite. These students stumbled upon methods that seem to be correct, but that we have not verified.

**Part 4 (Duration – 30 minutes):**

1. Group students in pairs.
2. Review definitions of altitude and angle bisector in a triangle.
3. Distribute Student Activity Sheet 4 "More About medians." This will lead students through an investigation to discover the following little known property of medians.

    The medians of any triangle form a new triangle whose area is ¾ the area of the original triangle. (If this part of the activity is presented after students have studied similar triangles, the statement can be expanded to state that the two triangles are similar.)

**Evaluation:**

- The teacher should circulate constantly to observe student progress and performance and to assist when problems arise.
- The teacher should give feedback to students as they present their computer solutions to the class.
- The teacher should review student conjectures and student work recorded on the Student Activity Sheets.
- The teacher should assess the level of understanding as demonstrated in student responses on "Activity Summary Sheet."

**Extension/Follow Up:**

- Students should be encouraged to research centers of mass in physics textbooks.
- The School's physics teacher can be invited into the geometry class as a guest speaker to give students more background on centers of mass (for objects with uniform density or varying density).
- If students have previously studied coordinate geometry, students can be challenged to find the coordinates of the centroids of various triangles that have been placed in the coordinate plane.
- Students can investigate relationships involving altitudes and angle bisectors that are similar to those discovered about medians.

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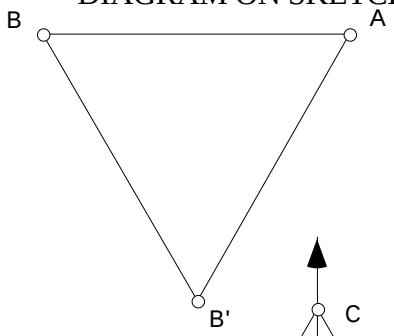
# Constructing Geometric Figures

Directions: Select three of the following figures and construct them using the *Geometer's Sketchpad*. Record at least two different methods of construction. The construction of the equilateral triangle has been demonstrated below.

## Isosceles triangle, parallelogram, rhombus, rectangle, square, trapezoid, isosceles trapezoid

Construction of Equilateral Triangle

DIAGRAM ON SKETCHPAD

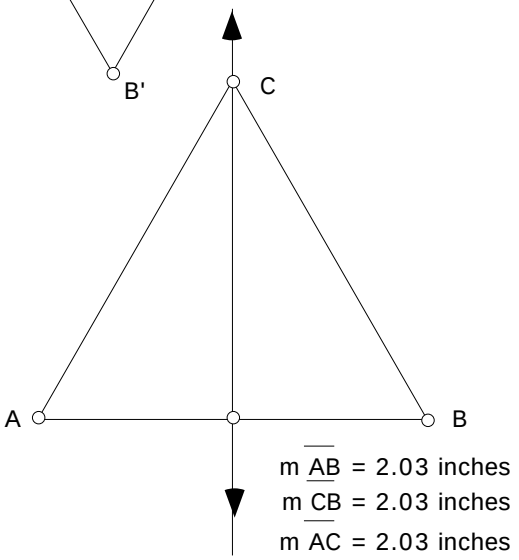



### Method 1

Steps:
1. Construct segment AB.
2. Rotate segment AB 60° about center point A. Image is segment BB'.
3. Construct segment BB'.

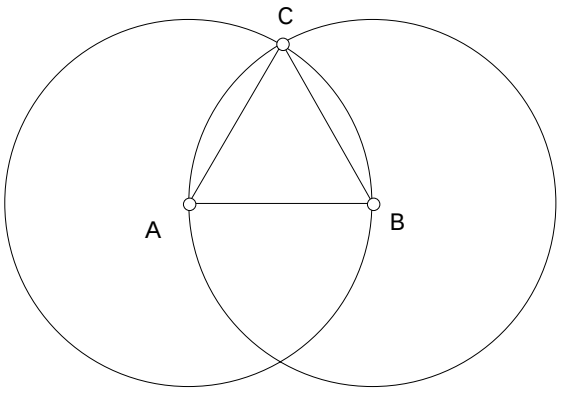
### Method 2

Steps:
1. Construct segment AB.
2. Construct perpendicular bisector of segment AB.
3. Measure length of segment AB.
4. Construct point C on the perpendicular bisector.
5. Construct segment AC and segment BC.
6. Measure length of segment AC and segment BC .
7. Slide point C along the perpendicular bisector until all three segments are equal to each other.




### Method 3

Steps:
1. Construct segment AB.
2. Construct a circle with center A and radius AB.
3. Construct a circle with center B and radius AB.
4. Construct the point of intersection of the two circles. This is point C.
5. Construct segments CA and CB.



I. Construction of_____

Method 1 DIAGRAM ON SKETCHPAD

Steps:

Method 2 DIAGRAM ON SKETCHPAD

Steps:

Method 3 DIAGRAM ON SKETCHPAD

Steps:

II. Construction of_____

Method 1 DIAGRAM ON SKETCHPAD

Steps:

Method 2 DIAGRAM ON SKETCHPAD

Steps:

Method 3 DIAGRAM ON SKETCHPAD

Steps:

III. Construction of_____

Method 1 DIAGRAM ON SKETCHPAD

Steps:

Method 2 DIAGRAM ON SKETCHPAD

Steps:

Method 3 DIAGRAM ON SKETCHPAD

Steps:

# The Balancing Act

## starring....The Triangle

**Objective:** Investigate the centroid in a triangle. Make conjectures about the lengths of segments and areas of triangles formed.

**Directions:**

1. Construct a scalene triangle.
2. Construct a median from each vertex.
3. Construct the point of intersection of the three medians. Record diagram below. Label all points.
4. Measure the lengths of the segments formed by the centroid on each median. Record data below.
5. Measure the areas of the triangles formed by the medians. Record data below.
6. Drag the vertices of the triangle to construct different triangles. Record data for at least two more triangles below.

## Diagram:

**Data:**

Triangle 1

| Segment | Length |
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| Triangle | Area |
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Triangle 2

| Segment | Length |
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| Triangle | Area |
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Triangle 3

| Segment | Length |
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| Triangle | Area |
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**Conjectures:**

# The Balancing Act
## Starring…The Quadrilateral

**Objective:** Investigate the location of the centroid in a quadrilateral.

**Procedure: Part 1**

1. Construct a rectangle on a piece of cardboard.
2. State a hypothesis about where the centroid is located.

_____

_____

3. Use your hypothesis to construct the centroid. Describe how you did this.

_____

_____

4. Place the rectangle on the eraser end of a pencil at this point and test if the rectangle balances.
5. Describe the results. If the rectangle doesn't balance at this point, move the pencil until it does balance. Describe the location of this point.

_____

_____

6. Put a small hole in the cardboard rectangle as close as possible to one of the vertices. Drop a plumb line. Do the same thing at another vertex. What is the significance of the point of intersection of the two plumb lines?

_____

_____

## Diagram:

**Procedure: Part 2**

1. Construct a rhombus on a piece of cardboard.

2. State a hypothesis about where the centroid is located.

_____

_____

3. Use your hypothesis to construct the centroid. Describe how you did this.

_____

_____

4. Place the rhombus on the eraser end of a pencil at this point and test if the rhombus balances.

5. Describe the results. If the rhombus doesn't balance at this point, move the pencil until it does balance. Describe the location of this point.

_____

_____

6. Put a small hole in the cardboard rhombus as close as possible to one of the vertices. Drop a plumb line. Do the same thing at another vertex. What is the significance of the point of intersection of the two plumb lines?

_____

_____

## Diagram:

**Procedure: Part 3**

1. Construct a kite on a piece of cardboard.

2. State a hypothesis about where the centroid is located.

_____

_____

3. Use your hypothesis to construct the centroid. Describe how you did this.

_____

_____

4. Place the kite on the eraser end of a pencil at this point and test if the kite balances.

5. Describe the results. If the kite doesn't balance at this point, move the pencil until it does balance. Describe the location of this point.

_____

_____

6. Put a small hole in the cardboard kite as close as possible to one of the vertices. Drop a plumb line. Do the same thing at another vertex. What is the significance of the point of intersection of the two plumb lines?

_____

_____

7. Using the Geometer Sketchpad, construct a kite and its centroid. Describe the procedure you used to construct the centroid below.

_____

_____

## Diagram:

**Procedure: Part 4**

1. Construct a quadrilateral on a piece of cardboard.

2. State a hypothesis about where the centroid is located.

_____

_____

3. Use your hypothesis to construct the centroid. Describe how you did this.

_____

_____

4. Place the quadrilateral on the eraser end of a pencil at this point and test if the quadrilateral balances.

5. Describe the results. If the quadrilateral doesn't balance at this point, move the pencil until it does balance. Describe this point.

_____

_____

6. Put a small hole in the cardboard quadrilateral as close as possible to one of the vertices. Drop a plumb line. Do the same thing at another vertex. What is the significance of the point of intersection of the two plumb lines?

_____

_____

7. Using the Geometer Sketchpad, construct a quadrilateral and its centroid. Describe the procedure you used to construct the centroid below.

_____

_____

## Diagram:

# MORE ABOUT MEDIANS

Instructions: Use *The Geometer's Sketchpad* to answer each of the following questions.

1. Construct an equilateral triangle ABC.
2. Construct the three medians in triangle ABC
3. Find the length of each median of ΔABC and place values in the chart.
4. Find the perimeter of ΔABC and find the area of ΔABC and place the values in the chart.
5. Use the medians of ΔABC to construct a new triangle that has the three medians as its sides.
6. Find the area and perimeter of the triangle formed by the medians and place the values in the chart.
7. Construct a new ΔABC that is <u>not</u> congruent to the first one you constructed.
8. Repeat questions 2-6 using this new ΔABC.
9. By computing the ratios indicated in the chart, discover a relationship between the areas and /or perimeters of the triangles you constructed and the ones you constructed from their medians. Write a statement describing the relationship(s) you have discovered.
10. Repeat questions 2-9 for the <u>altitudes</u> of Δ ABC.
11. Repeat questions 2-9 for the <u>angle bisectors</u> of ΔABC.

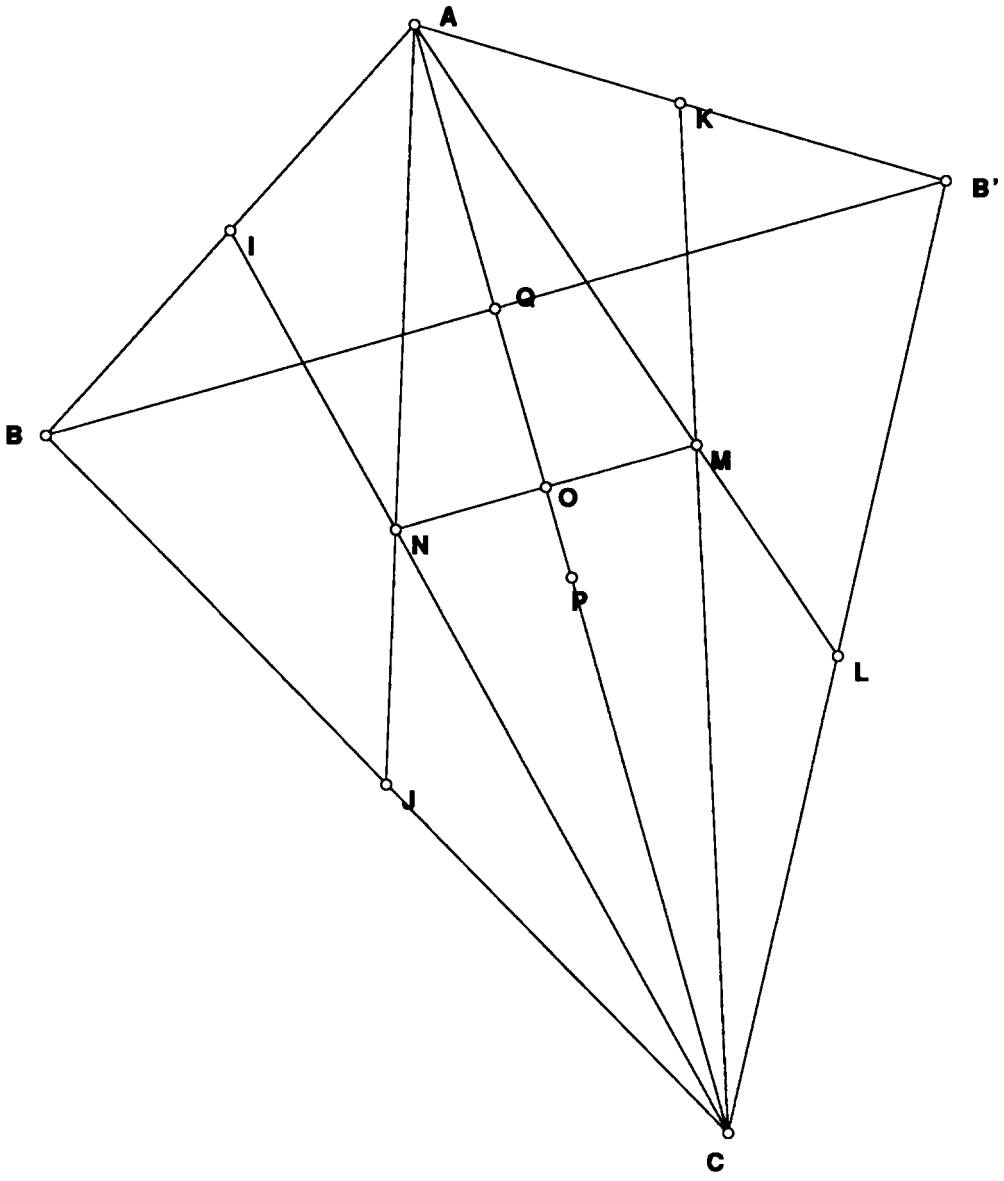
| lengths of medians of first ΔABC | Area of ΔABC | Area of median ΔABC | Ratio of Areas |
|---|---|---|---|
| | | | |
| | Perimeter of ΔABC | Perimeter of median ΔABC | Ratio of Perimeters |
| | | | |
| lengths of medians of second ΔABC | Area of ΔABC | Area of median ΔABC | Ratio of Areas |
| | | | |
| | Perimeter of ΔABC | Perimeter of median ΔABC | Ratio of Perimeters |
| | | | |

## Unverified Student Solution

PO = 0.29 inches

OQ = 0.58 i

O = centroid



**Unverified Student Solution**

m $\overline{AB}$ = 9.7 cm
XW = 0.4 cm
WC = 0.8 cm
W = centroid